Pertemuan 6
Alin 2017
Bilqis

Cross Product

Persamaan bidang

TUJUAN INSTRUKSIONAL KHUSUS

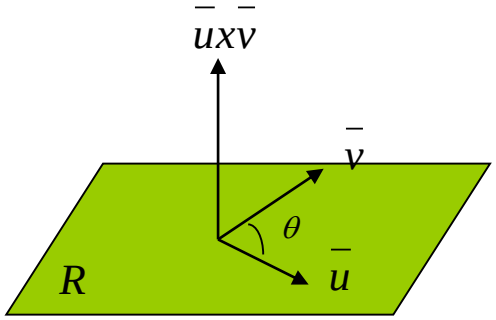
Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Dapat menghitung perkalian silang dari dua vektor dan mengetahui contoh aplikasinya

3.4 **HASIL** Kali Silang **-> Cross**

Hasil **-> Vector**

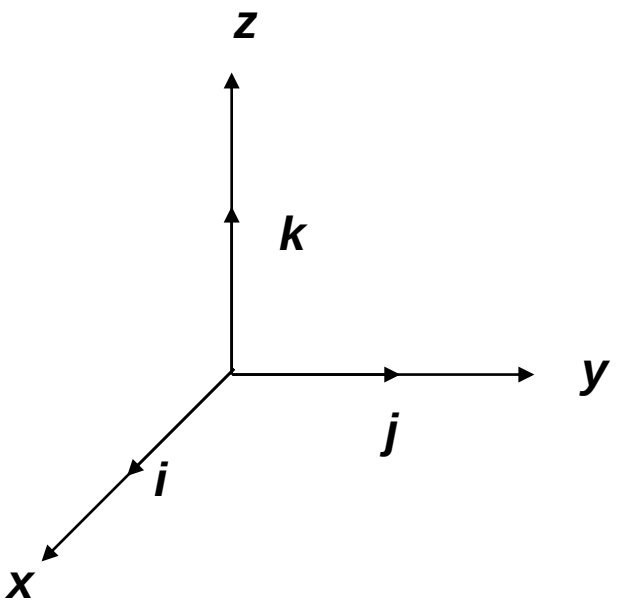
Berbeda denan hasil kali titik, hasil ->skalar



Menggunakan >>
kaidah tangan kanan

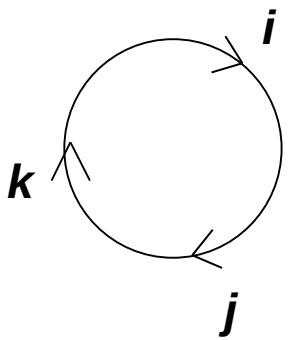
$$\left.\begin{array}{l}\bar{u}\ x\ \bar{v} \perp \bar{v} \\ \bar{u}\ x\ \bar{v} \perp \bar{u}\end{array}\right\} \text{p}ada \text{ bidang R}$$

$$\begin{bmatrix} i\ (1,0,0) \\ j\ (0,1,0) \\ k\ (0,0,1) \end{bmatrix}$$



$$\bar{v}\begin{cases} (v_1, v_2, v_3) \\ v_1 i + v_2 j + v_3 k \end{cases}$$

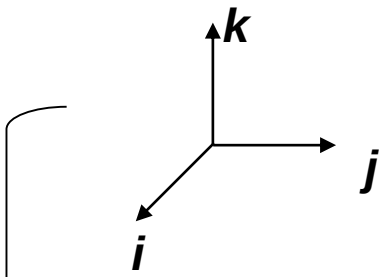
$$\bar{v}\begin{cases} (2,-3,4) \\ 2i - 3j + 4k \end{cases}$$



$$\begin{cases} searah\ jarum\ jam & = + \\ berlawanan\ \ jarum\ \ jam & = - \end{cases}$$

i x j = k

j x I = -k



Kaidah tangan kanan

$$\bar{u} = (u_1, u_2, u_3)$$

$$\bar{v} = (v_1, v_2, v_3)$$

$\bar{u} \times \bar{v}$

4 (w_1, w_2, w_3)

3 $(u_2.v_3 - u_3.v_2, u_3.v_1 - u_1.v_3, u_1.v_2 - u_2.v_1)$

2 $\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$

$$\left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right)$$

1 $$\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} i - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} j + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} k$$

Contoh 12

$$\| \bar{u} \times \bar{v} \| = \begin{bmatrix} \sqrt{w_1^2 + w_2^2 + w_3^2} \\ \|\bar{u}\|.\|\bar{v}\|.\sin\theta \end{bmatrix}$$

Contoh 12

Carilah ***u x v*** dimana *u = (1,2,-2)* dan *v = (3,0,1)*

Pemecahan :

$$\bar{u}\ x\ \bar{v} \neq \bar{v}\ x\ \bar{u} \rightarrow beda\ arah$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 3 & 0 & 1 \end{bmatrix}$$

$$u\ x\ v = \left(\begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \right)$$

$$= (2, -7, -6)$$

Cari juga :

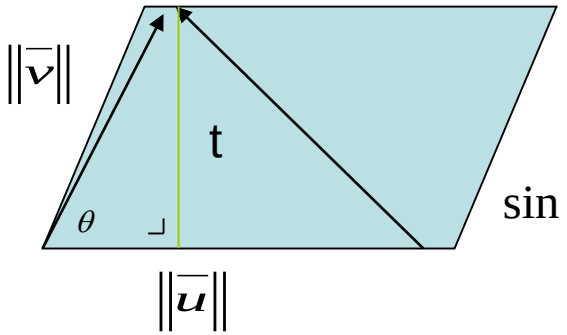
1. u.v =
2. V x u =
3. | u x v | =

☺ Luas Jajaran Genjang

$$Ljg = alas \; x \; tinggi$$

$$= \|\bar{u}\| \, \|\bar{v}\| \sin\theta$$

$$= \|\bar{u} \; x \; \bar{v}\|$$



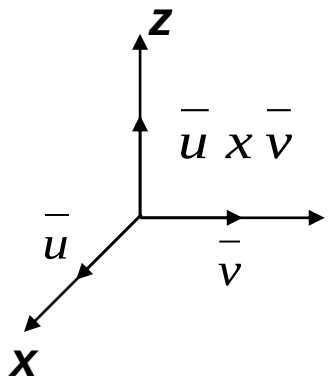
$$\sin\theta = \frac{t}{\|\bar{v}\|}$$

Luas Δ = $\frac{1}{2}$. Ljg

Contoh 15

$\bar{u} \times \bar{v}$ ⟶ Bebas Koordinat

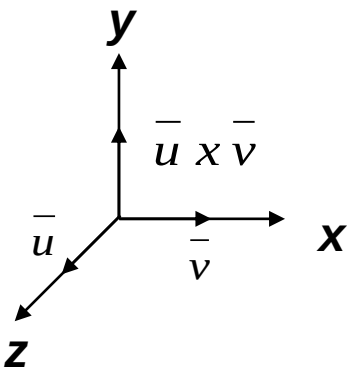
⟶ Meskipun koordiat diganti tetapi, arah vektor $\bar{u} \; x \; \bar{v}$ tetap sama



$\bar{v}$ = (1, 0, 0)

$\bar{u}$ = (0, 1, 0)

$\bar{u} \; x \; \bar{v}$ = (0, 0, 1)



$\bar{v}$ = (0, 0, 1)

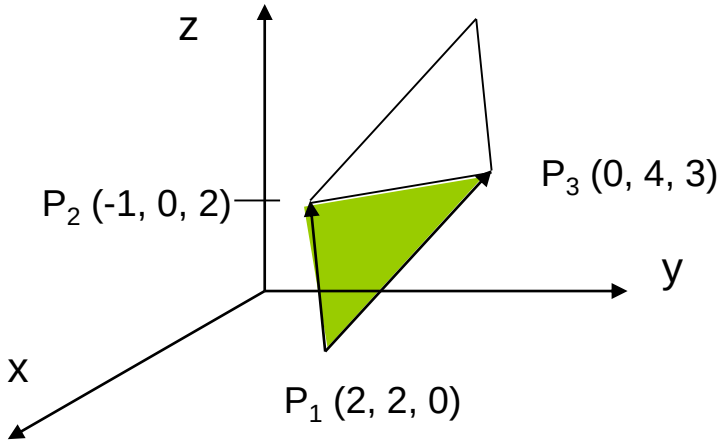
$\bar{u}$ = (1, 0, 0)

$\bar{u} \; x \; \bar{v}$ = (0, 1, 0)

Contoh 15:

Carilah luas segitiga yang ditentukan oleh titik – titik P_1 (2, 2, 0), P_2 (-1, 0, 2), P_3 (0, 4, 3).

Pemecahan. Luas segitiga tersebut adalah ½ luas jajaran genjang yang ditentukan oleh vektor – vektor $\overrightarrow{P_1P_2}$ & $\overrightarrow{P_1P_3}$ (gambar 3.28).



Gambar 3.28

Dengan menggunakan metode yang kita bahas pada contoh 2 dari bagian 3.1, maka $\overrightarrow{P_1P_2}$ (-3, -2, 2) dan $\overrightarrow{P_1P_3}$ = (-2, 2, 3). Jelaslah bahwa

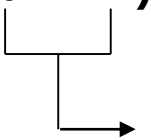
$$\overrightarrow{P_1P_2} \text{ X } \overrightarrow{P_1P_3} = (-10, 5, -10)$$

Dan konsekuensinya maka

$$\mathbf{A = \tfrac{1}{2}\| \overrightarrow{P_1P_2} \text{ X } \overrightarrow{P_1P_3} \| = \tfrac{1}{2}(15) = 15/2}$$

Rumus :

1.) $\bar{u} \cdot (\bar{u} \times \bar{w}) = 0$



Vektor yang $\perp$ terhadap $\bar{u}$

2.) $\bar{v} \cdot (\bar{u} \times \bar{v}) = 0$

3.) $\| \bar{u} \times \bar{v} \|^2 = \| \bar{u} \|^2 \cdot \| \bar{v} \|^2 - (\bar{u} \cdot \bar{v})^2$

4.) $\bar{u} \times \bar{v} = - (\bar{v} \times \bar{u})$

5.) $\bar{u} \times (\bar{v} + \bar{w}) = (\bar{u} \times \bar{v}) + (\bar{u} \times \bar{w})$

6.) $(\bar{u} + \bar{v}) \times \bar{w} = (\bar{u} \times \bar{v}) + (\bar{u} \times \bar{w})$

7.) $k (\bar{u} \times \bar{v}) = (k (\bar{u})) \times \bar{v} = \bar{u} \times (k \bar{v})$

8.) $\bar{u} \times 0 = 0 \times \bar{u} = 0$

9.) $\bar{u} \times \bar{u} = 0$

Example 2, Consider the Vectors

$$u = (1, 2, -2) \text{ and } v = (3, 0, 1)$$

In example 1 we showed that

$$(\bar{u} \times \bar{v}) \cdot u = \emptyset$$

$$u \times v = (2, -7, -6)$$

$$(\bar{u} \times \bar{v}) \cdot v = \emptyset$$

Since

$$u \cdot (u \times v) = (1)(2) + (2)(-7) - (2)(-6) = 0$$

And

$$v \cdot (u \times v) = (3)(2) + (0)(-7) - (1)(-6) = 0$$

u X v is orthogonal to both u and v as guaranteed by theorhem 3.4.1

$$u \cdot v = 1.3 + 2.0 + -2.1$$

$$= 3 + 0 - 2$$

$$= 1$$

$$v \times u = \begin{vmatrix} 3 & 0 & 1 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \left(\begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}, -\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}, \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} \right)$$

$$= (-2, 7, 6)$$

$$(u \times v) \cdot u = \emptyset$$

$$(2, -7, -6) \cdot (1, 2, -2) = 2 - 14 + 12$$

$$= \emptyset$$

$$|u \times v| = \sqrt{2^2 + (-7)^2 + (-6)^2}$$

$$= \sqrt{4 + 49 + 36}$$

Definisi:

Jika **u**, **v**, dan **w** merupakan vektor di ruang-3, maka **u** • (**v** × **w**) disebut sebagai hasil kali skalar ganda tiga (*scalar triple product*) dari **u**, **v**, dan **w**

$$\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Teorema 3.4.5:

Jika **u**, **v**, dan **w** merupakan vektor di ruang-3, dan ketiga titik inisialnya berimpit, maka ketiga vektor tersebut terletak dalam satu bidang datar jika dan hanya jika

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 0$$

$(u \times v) \cdot w = 0$

$(u \times w) \cdot v = 0$

$(v \times w) \cdot u = 0$

Contoh (diambil dari Exercise Set 3.4 no. 11b):

Tentukan apakah **u** = (5, – 2, 1), **v** = (–4, –1, 1), **w** = (1, –1, 0) terletak pada bidang datar yang sama jika diposisikan sedemikian sehingga titik-titik inisialnya berimpit.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 5 & -2 & 1 \\ -4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 5(0+1) + 2(0-1) + 1(4+1) = 8$$

Jadi ketiga vektor tersebut tiak terletak dalam bidang datar yang sama

Garis dan Bidang di Ruang-3

Bab 3.5

TUJUAN INSTRUKSIONAL KHUSUS

Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Dapat menyelesaikan permasalahan garis dalam ruang-2 dan ruang-3 dengan pendekatan vektor

- Bentuk umum persamaan bidang yang mempunyai <u>vektor normal</u> $\bar{n} = (a, b, c)$

  vektor yang tegak lurus thdp bidang

  Diket: - $P_0 (x_0, y_0, z_0)$

  - $\bar{n} = (a, b, c)$

  => $ax + by + cz + d = 0$ ⟶ persamaan bidang

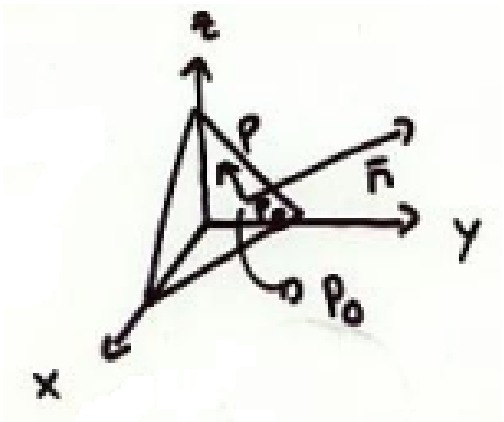
  contoh :

  Catatan ➔

  x, y, z → tetap

  x_0, y_0, z_0 } angka

  a, b, c }



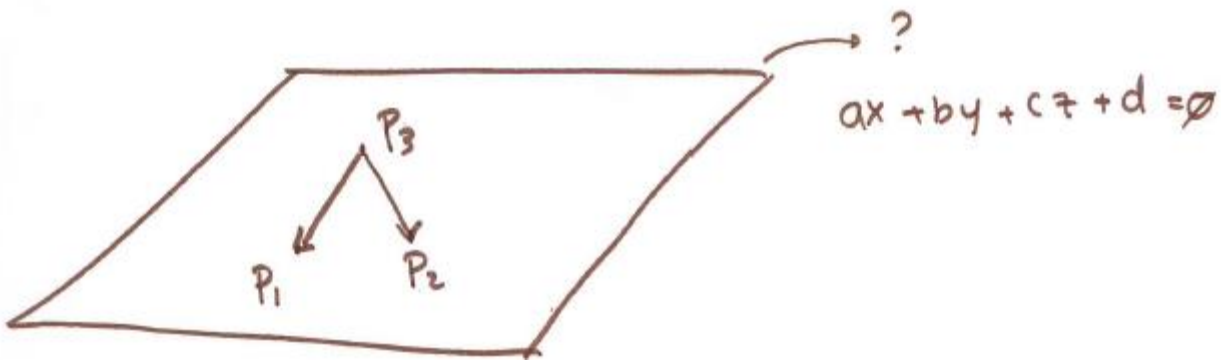
$P = (x, y, z)$

$P_0 = (x_0, y_0, z_0)$

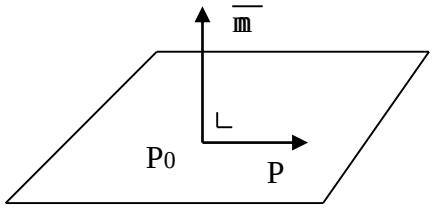
$\bar{n} = (a, b, c)$

$\overrightarrow{P_0 P} \perp \bar{n}$

$\overline{P_0 P} \cdot \bar{n} = 0$


?
ax + by + cz + d = 0
P3
P1
P2

3.5 Garis dan bidang di ruang 3



$\overline{P_0 P} \cdot \bar{n} = 0$

variabel ← P = (x, y, z)

angka { $P_0 = (x_0, y_0, z_0)$ → titik pada bidang

$\bar{n} = (a, b, c)$ → vektor normal / vektor yang ⊥ thdp bidang

Pers. bidang → $\overline{P_0 P} \cdot n = 0$

$(x - x_0 , y - y_0 , z - z_0) \cdot (a, b, c) = 0$

$a (x - x_0) + b (y - y_0) + c (z - z_0) = 0$ → bentuk titik normal

$ax + by + cz + (- a x0 - b y0 - c z0) = 0$

$ax + by + cz + \quad d \quad = 0$ → Pers. bidang

Contoh : 17 Cari persamaan bidang yang melewati (3, -1, 7)

dan $\perp$ thdp n = (4, 2, -5)

Jawab: $\overline{P_0 P} \cdot \overline{n} = 0$

$(x - x_0 , y - y_0 , z - z_0) \cdot (a, b, c) = 0$

$4 (x – 3) + 2 (y + 1) + -5 (z - 7) = 0$

$4x + 2y - 5z + (- 12 + 2 + 35) = 0$

$4x + 2y - 5z + 25 = 0$ → pers. bidang

bukti: kita masukkan P_0 = (3, -1, 7) ke dlm persamaan:

$\Rightarrow 4.3 + 2.-1 + 5.7 + 25 = 0$

$12 -2 – 35 + 25 = 0$

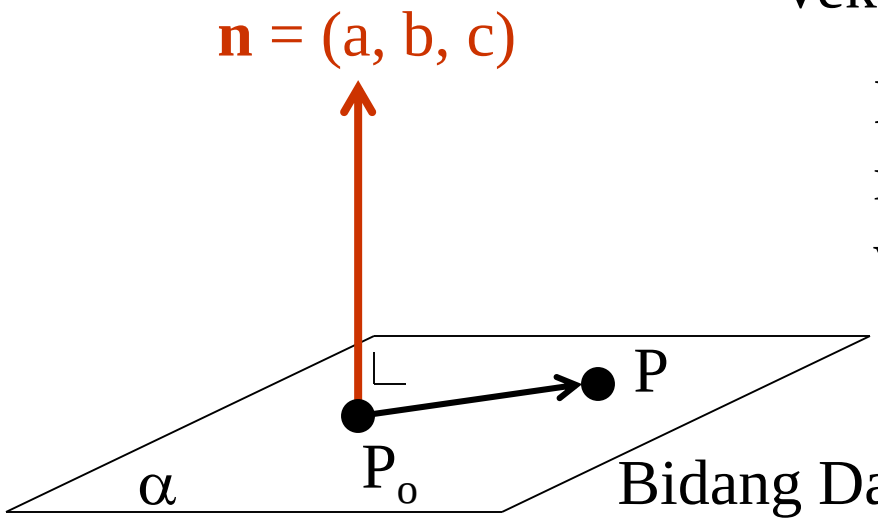
$0 = 0$ → terbukti

Bidang Datar:

Persamaan normal-titik (*point normal form*):

Titik $P_o(x_o,y_o,z_o)$ dan titik P(x, y, z) terletak di bidang datar α

Vektor normal **n** = (a, b, c) ortogonal terhadap bidang α



Vektor $P_oP = (x - x_o, y - y_o, z - z_o)$

Karena **n** ortogonal terhadap α, maka **n** juga ortogonal terhadap vektor P_oP, sehingga

$$\mathbf{n} \cdot P_oP = 0$$

Bidang Datar α dinyatakan dengan persamaan:

$$a(x - x_o) + b(y - y_o) + c(z - z_o) = 0$$

Bidang Datar:

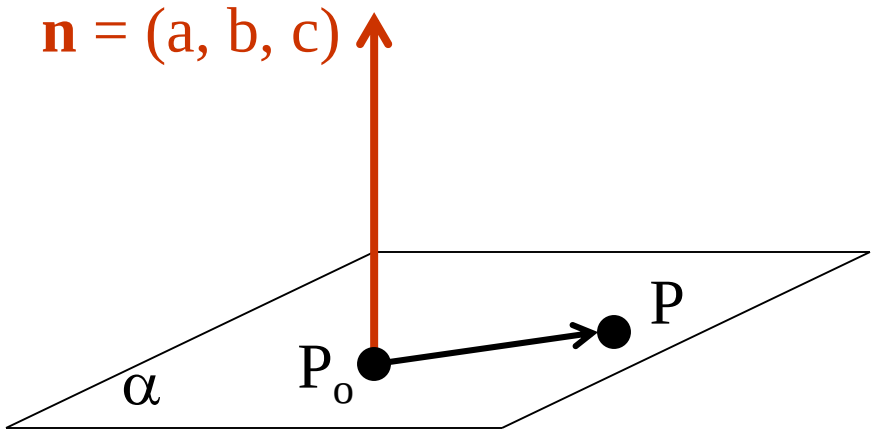
Bentuk umum Persamaan Bidang Datar:

Dari Persamaan Normal-titik (*point normal form*):

$$a(x - x_o) + b(y - y_o) + c(z - z_o) = 0$$

$$ax + by + cz + (\underbrace{- ax_o - by_o - cz_o}) = 0$$

$$ax + by + cz + \quad d \quad = 0$$



Bidang Datar α dinyatakan dengan persamaan :

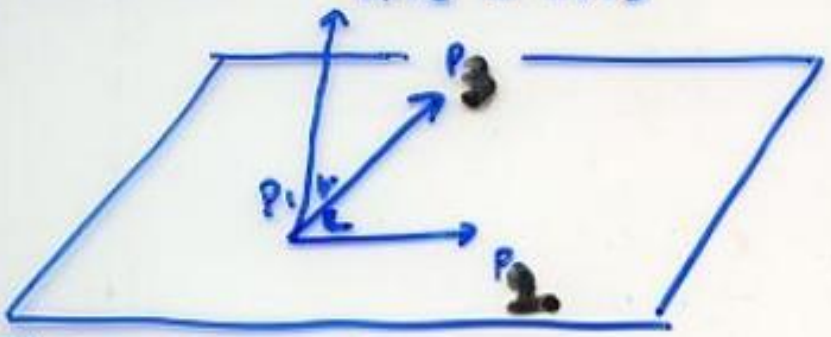
$$ax + by + cz + d = 0$$

Contoh 18

Cari pers bid. yg melalui $P_1(1,2,-1)$ $P_2(2,3,1)$ & $P_3(3,-1,2)$

$\overline{P_1P_2} \times \overline{P_1P_3}$

Jwb:



$\overline{P_1P_2}$ → pd bidang

$\overline{P_1P_3}$ → pd bidang

$\overline{P_1P_2} = (1, 1, 2)$

$\overline{P_1P_3} = (2, -3, 3)$

$\underbrace{\overline{P_1P_2} \times \overline{P_1P_3}}_{\bar{n}} = (9, 1, -5)$ → vektor normal yg ⊥ thp $\overline{P_1P_2}$ & ⊥ thp $\overline{P_1P_3}$

Pers. bidang → $\overline{P_0P} \cdot \bar{n} = 0$ → $P_0 = P_1 = (1,2,-1)$, $\bar{n} = (9,1,-5)$

$(x - x_0, y - y_0, z - z_0) \cdot (a, b, c) = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$9(x - 1) + 1(y - 2) + -5(z + 1) = 0$

$9x + y - 5z + (-9 - 2 - 5) = 0$

$9x + y - 5z - 16 = 0$

$\underline{\cup}$ P_0 dpt di isi $\underline{\cup}$ P_1 $\underline{\text{or}}$ P_2 $\underline{\text{or}}$ P_3, krn hasilnya sama $\rightarrow$ terletak pd. bid

ex $\rightarrow$

$P_0 = P_2 = (2, 3, 1)$

$\bar{n} = (9, 1, -5)$

$\Rightarrow 9(x-2) + 1(y-3) - 5(z-1) = 0$

$9x + y - 5z + (-18 - 3 + 5) = 0$

$9x + y - 5z - 16 = 0$ $\rightarrow$ sama spt di atas

ex $\rightarrow$

$P_0 = P_3 = (3, -1, 2)$

$\bar{n} = (9, 1, -5)$

$\Rightarrow 9(x-3) + 1(y+1) - 5(z-2) = 0$

$9x + y - 5z + (-27 + 1 + 10) = 0$

$9x + y - 5z - 16 = 0$

Contoh soal No. 1

Cari pers. bidang yg melalui $P_1(-3, 2, 0)$ $P_2(0, -1, 2)$ dan $P_3(5, 1, 3)$. (Titik pusat P_2) & $P_0 = P_1$

Pers. bidang $\rightarrow$ $\overline{P_0 P} \cdot \bar{n} = 0$

(2) $P_0 = (-3, 2, 0)$

$P = (x, y, z)$

$n = P_2P_1 \times P_2P_3$

$P_2P_1 = P_1 - P_2 = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$

(2) $= \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

$P_2P_3 = P_3 - P_2 = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$

(2) $= \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$

(2) $P_2P_1 \times P_2P_3 = \begin{bmatrix} -3 & 3 & -2 \\ 5 & 2 & 1 \end{bmatrix}$

(2) $= \left(\begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix}, - \begin{vmatrix} -3 & -2 \\ 5 & 1 \end{vmatrix}, \begin{vmatrix} -3 & 3 \\ 5 & 2 \end{vmatrix} \right)$

(2) $P_2P_1 \times P_2P_3 = \bar{n} = (7, -7, -21)$

(2) $\overline{P_0 P} \cdot \bar{n} = 0$

(2) $\left((x+3), (y-2), (z-0)\right) \cdot (7, -7, -21) = 0$

(2) $7(x+3) - 7(y-2) - 21z = 0$

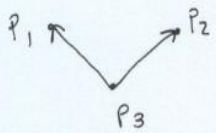
(2) $7x - 7y - 21z + 35 = 0$

Contoh Soal No. 2

Carilah luas segitiga yang ditentukan oleh titik - titik $P_1 = (1, -2, -3)$ $P_2 = (5, 4, -3)$ $P_3 = (5, -1, 7)$ (titik pusatnya adalah P_3)

jawab :

luas $\Delta = \frac{1}{2} \left| P_3P_1 \times P_3P_2 \right|$



$P_3P_1 = P_1 - P_3 = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ 7 \end{pmatrix}$

(2) $= \begin{pmatrix} -4 \\ -1 \\ -10 \end{pmatrix}$

$P_3P_2 = P_2 - P_3 = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ 7 \end{pmatrix}$

(2) $= \begin{pmatrix} 0 \\ 5 \\ -10 \end{pmatrix}$

(2) $P_3P_1 \times P_3P_2 = \begin{bmatrix} -4 & -1 & -10 \\ 0 & 5 & -10 \end{bmatrix}$

$= \left(\begin{vmatrix} -1 & -10 \\ 5 & -10 \end{vmatrix}, - \begin{vmatrix} -4 & -10 \\ 0 & -10 \end{vmatrix}, \begin{vmatrix} -4 & -1 \\ 0 & 5 \end{vmatrix} \right)$

(2) $= (60, -40, -20)$

(2) $\left| P_3P_1 \times P_3P_2 \right| = \sqrt{60^2 + (-40)^2 + (-20)^2}$

$= \sqrt{5600}$

(2) $= 74,83$

luas $\Delta = \frac{1}{2} \left| P_3P_1 \times P_3P_2 \right|$

(2) $= \frac{1}{2} \cdot 74,83$

(2) $= 37,42$

Contoh Soal No.3

Apakah $A = (6, 7, -1)$ $B = (-1, 2, 4)$ $C = (7, -3, 0)$ terletak pada bidang datar yg sama jika di porisikan sedemikian sehingga titik - titik inisialnya berimpit

Jawab

Cek apakah $A.(B \times C) = 0$

(3) $A.(B \times C) = \begin{vmatrix} 6 & 7 & -1 \\ -1 & 2 & 4 \\ 7 & -3 & 0 \end{vmatrix}$

(3) $= 6.\begin{vmatrix} 2 & 4 \\ -3 & 0 \end{vmatrix} - 7\begin{vmatrix} -1 & 4 \\ 7 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & 2 \\ 7 & -3 \end{vmatrix}$

$= 6.(12) - 7(-28) - 1(3-14)$

(3) $= 279$

(3) krn $\neq 0$ mk A B C tidak terletak dlm bidang datar yg sama

Contoh Soal No. 4

Carilah luas segitiga yang ditentukan oleh titik-titik P1(-2, 3, 4), P2(7, -1, 3) dan P3(4, 7, 2). Titik pusat adalah P2.

P1 =	-2		P2P1	= P1 - P2			
	3			x =	-9		nilai 1
	4			y =	4		nilai 1
				z =	1		nilai 1
P2 =	7						
	-1						
	3						
P3 =	4		P2P3	= P3 - P2			
	7			x =	-3		nilai 1
	2			y =	8		nilai 1
				z =	-1		nilai 1
p2p1 x p2p3		-9	4	1			
		-3	8	-1			
	x =	-12					
	y =	-12		nilai 3			
	z =	-60		nilai 3			
luas segitiga		31.17691		nilai 3			

Contoh Soal No. 5

- Carilah persamaan bidang yang melalui P1(-3, 3, 6), P2(2, -4, 6) dan P3(3, 4, 2). Titik pusat adalah P3

P1	-3		p3p1	= P1 - P3			
	3			x =	-6		nilai 1
	6			y =	-1		nilai 1
				z =	4		nilai 1
P2	2						
	-4						
	6						
P3	3		p3p2	= P2 - P3			
	4			x =	-1		nilai 1
	2			y =	-8		nilai 1
				z =	4		nilai 1
p3p1 x p3p2		-6	-1	4			
		-1	-8	4			
vektor normal							
	x =	28		nilai 3			
	y =	20		nilai 3			
	z =	47		nilai 3			
		PoP . Normal = 0					
po = p1							
po	x =	-3		a =	28		
	y =	3		b =	20		
	z =	6		c =	47		
	a(x - xo) + b (y -yo) + c (z - zo) = 0				nilai 3		
	28(x +3) + 20 (y - 3) + 47 (z - 6) = 0				nilai 3		
	28 x + 84 + 20 y - 60 + 47 z - 282 = 0				nilai 3		
	28 x + 20 y + 47 z - 258 = 0				nilai 3		

Contoh soal No. 6

- Carilah luas segitiga yang ditentukan oleh titik-titik P1 (4, -8, 2), P2(6, 3, -5) dan P3 (8, 2, 5).
- Catt : titik pusat adalah P3 maka perkalian crossnya adalah P3P1 x P3P2

P1 =	**4**		**P3P1**	**= P1 - P3**			
	-8			x =	-4		nilai 1
	2			y =	-10		nilai 1
				z =	-3		nilai 1
P2 =	6						
	3						
	-5						
P3 =	8		P3P2	= P2 - P3			
	2			x =	-2		nilai 1
	5			y =	1		nilai 1
				z =	-10		nilai 1
p3p1 x p3p2		-4	-10	-3			
		-2	1	-10			
	x =	103					
	y =	-34		nilai 3			
	z =	-24		nilai 3			
luas segitiga		55,55		nilai 3			

- Tugas Kelompok ➔
  - cari 2 soal dan jawaban di internet yang berhubungan dengan materi ppt ini
  - Tulis alamat internetnya
  - Di kirim ke elearning, terakhir ➔
    - Minggu depan
- Format ➔ subject ➔
  - Alin-B-melati
  - Bentuk ➔ ppt ➔ informasi nama kelompok + anggota